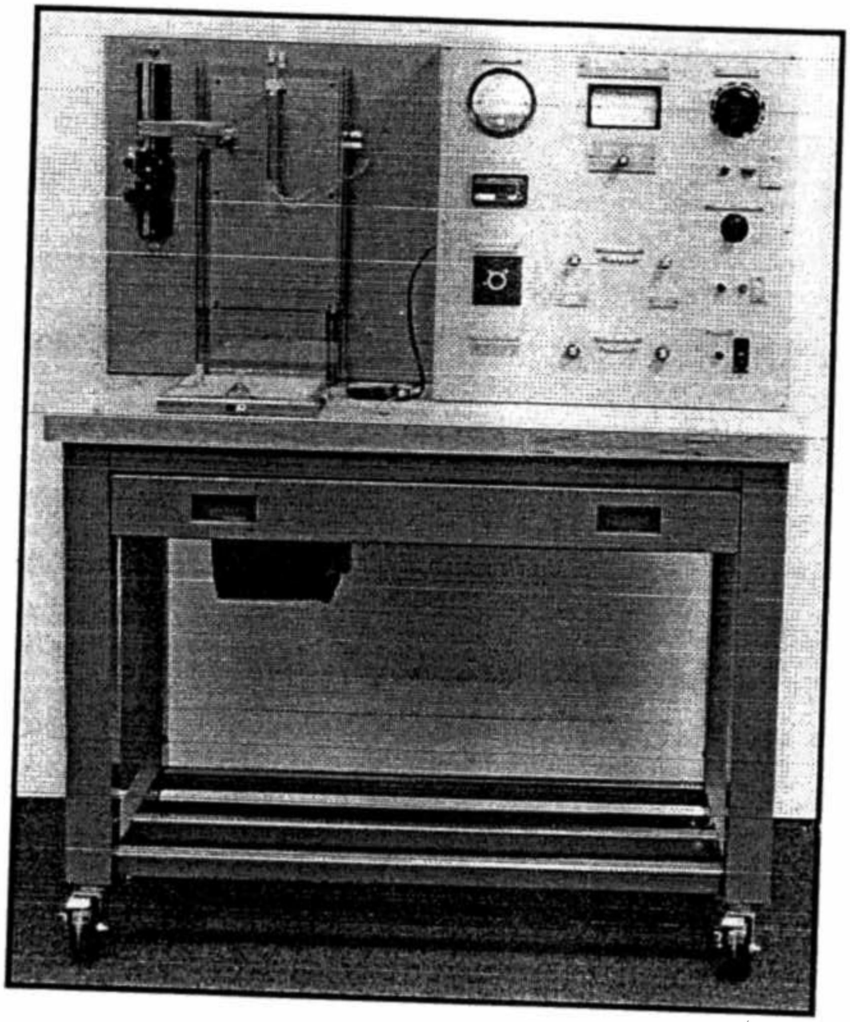
Bulletin 688-201

OPERATING INSTRUCTIONS

FOR THE



HAMPDEN MODEL H-6882

Convection Heat Transfer Demonstrator

SAFETY

READ INSTRUCTIONS COMPLETELY BEFORE STARTING TRAINER

Normal operation of the trainer is not considered hazardous. However, the RECOMMENDED PROCEDURE SHOULD BE FOLLOWED to be sure that the classroom instruction is performed under the safest possible conditions. If the operator knows and understands the construction and operation of the parts in the system, it will help one to operate it safely.

THE OPERATOR SHOULD ALWAYS BE ALERT to experimental procedures which may be a hazard to the operator or be injurious to the equipment. Every control device and switch has a specific operational application. Be certain that all connections and control settings are carefully managed. NO SETTINGS SHOULD BE MADE INDISCRIMINATELY.

STUDENT USE OF THE TRAINER SHOULD ALWAYS BE SUPERVISED. Even the most experienced student never operate it while alone.

Always have good ventilation and good lighting when operating the system.

Instruments and equipment used in testing, while durable, are sensitive to abuse. When connecting an electrical instrument into a circuit, make sure that the instrument and its settings are within the voltage and current range which may be applied to the instrument. This will protect the trainer, the instrument, and the operator.

Use extreme CAUTION when making electrical measurements. Remember, it is too late to learn that a circuit is live after one has touched it. Be certain that the operator knows if the trainer is on or off at all times. Never handle live circuits when in contact with pipes, other wires, or damp floors.

Keep the floor clean of debris — oil, water, or other slippery material.

An electrical short across a ring or wristwatch can cause a severe burn. It is best to remove all watches and jewelry when working on electrical equipment.

Always disconnect the electrical power source before isolating any component, electrical or otherwise, from the trainer system. Lock the switches open to prevent someone from closing them during demonstration or test procedure.

LIST OF SYMBOLS

A	Area (m^2)
C_f	Friction coefficient
$C_p(c)$	Specific heat kJ/kg-°C)
d, D	Diameter (m)
g	Gravity (m/s^2)
h	Convective heat transfer coefficient (W/m^2-°C)
k	Thermal conductivity (W/m-°C)
l, L	Length (m)
m	Mass (kg)
$\dot{m}$	Mass flow rate (kg/s)
p, P	Pressure (N/m^2)
q	Heat transfer rate (kJ/s)
$\dot{q}_w$	Heat flux, (kJ/m^2-s)
$\dot{q}$	Heat generated per unit volume, (kJ/m^3-s)
Q	Heat (kJ)
r, R	Radial distance (m)
t, T	Temperature (°C)
U	Overall heat transfer coefficient (W/m^2-°C)
u, U	Velocity (m/s)
v, V	Velocity (m/s)
v	Specific volume (m^3/kg)
V	Volume (m^3)
X, Y, Z	Cartesian coordinates

Greek Letters

$\alpha = k/\rho c_p$	Molecular thermal diffusivity (m^2/s)
β	Volume coefficient of expansion (1/K)
ρ	Density (kg/m^3)
μ	Dynamic viscosity (kg/m-s)
$\nu = \mu/\rho$	Kinematic viscosity (m^2/s)
π	Pi
η	Similarity variable
θ	Non-dimensional temperature distribution
Ψ	Stream function

Dimensionaless Groups

$Gr = g\beta x^3 \frac{(t_o - t_\infty)}{\nu^2}$	Grashof number
$Nu = hx/k$	Nusselt number
$Pr = C_p \mu/k$	Prandtl number
$R_e = \rho\ ux/\mu$	Reynolds number
$St = \frac{Nu}{RePr} = \frac{h}{\rho\ c_p\ u_\infty}$	Stanton number
$\frac{C_f}{2} = StPr^{2/3}$	Friction Coefficient

Subscripts

d	based on diameter
f	film
i	inside
L	based on lengths
m	mean
o	initial, surface
r	at a specific radial distance
x	local position in the x-direction
w	wall conditions
∞	free stream conditions

SECTION 1

CONVENTION HEAT TRANSFER

When a fluid moves past a boundary, there is a velocity gradient normal to the boundary surface (for a two dimensional fluid flow). This velocity gradient arises from the shear stresses acting on the fluid. In general, the velocity gradient is confined within a small distance of the boundary surface and is called the boundary layer.

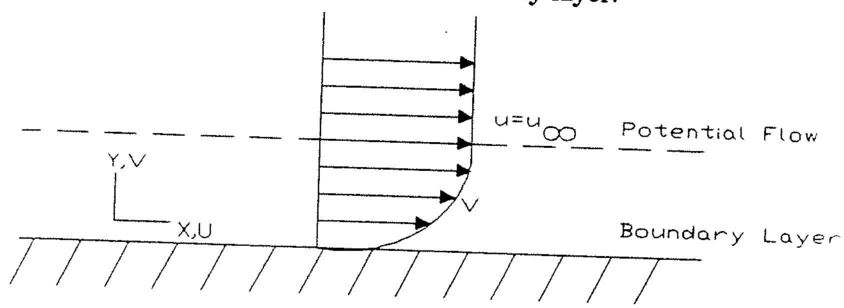


Figure 1-1 Boundary Layer on an External Surface

The three basic equations governing the boundary layer in two dimensional, differential form are:

Continuity $$\frac{\delta}{\delta x}(\rho u) + \frac{\delta}{\delta y}(\rho v) = o \qquad (1\text{-}1)$$

Momentum $$\rho u\frac{\delta u}{\delta x} + \rho v\frac{\delta u}{\delta y} + \frac{dP}{dx} = -\frac{\delta}{\delta y}\left\{\mu\frac{\delta u}{\delta y}\right\} - \rho g \qquad (1\text{-}2)$$

Energy $$\rho uc\frac{\delta t}{\delta x} + \rho vc\frac{\delta t}{\delta y} - \frac{\delta}{\delta t}\left\{k\frac{\delta t}{\delta y}\right\} - \left\{\frac{\delta u}{\delta y}\right\}^2 = o \qquad (1\text{-}3)$$

There are also two auxiliary equations to be considered

Hydrostatics $$\frac{dP}{dx} = -\rho_\infty g \qquad (1\text{-}4)$$

for vertical orientations

Volume coefficient of thermal expansion $$\beta \triangleq \frac{1}{v}\left(\frac{\delta V}{\delta T}\right)_p \qquad (1\text{-}5)$$

At this point in time, we shall make a few approximations.

(1) Neglect all variable property effects except for density in the momentum equation.

(2) Neglect the viscous dissipation term in the energy equation,

$$\mu \left(\frac{\delta u}{\delta y}\right)^2 = 0$$

(3) $$\beta = \frac{1}{V_\infty}\left(\frac{V - V_\infty}{T - T_\infty}\right) = \frac{1}{\rho}\left(\frac{\rho_\infty - \rho}{t - t_\infty}\right)$$

and for ideal gas, $\beta = \frac{1}{T_\infty}$

These are the governing equations, in differential form, used to solve for the heat transfer from a surface.

SECTION 2

FREE-CONVECTION HEAT TRANSFER

When the fluid flow past a boundary is induced by body forces, such as gravity, the resulting flow is called natural, or free convention. In contrast, if the fluid flow past a boundary is induced by external means, such as a mechanical blower, the resulting flow is called forced convection. In addition, if the free convective flow is on the order of the forced convective flow, one finds a mixed convective flow regime.

First, we shall investigate free convection flows over a vertical flat plate. One crucial assumption is that plate is semi-infinite, i.e. the flow is not affected by any boundaries. For a solid body orientated vertically in a gravitational field (see Figure 2-1A & B), the following boundary conditions are used

at $y = 0$	$u = v = 0$	$t = t_o(x)$
at $x = 0$	$u = 0$	$t = t_\infty$
as $y \rightarrow \infty$	$u \rightarrow 0$	$t \rightarrow t_\infty$

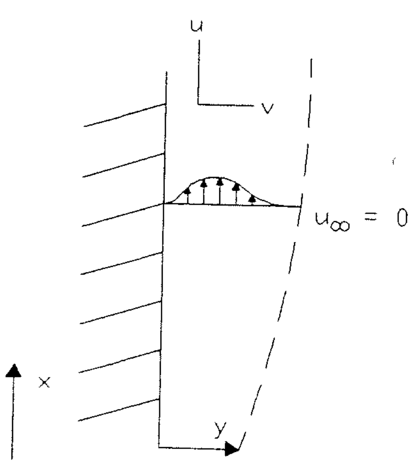


Figure 2-1A
Free convection momentum boundary layer

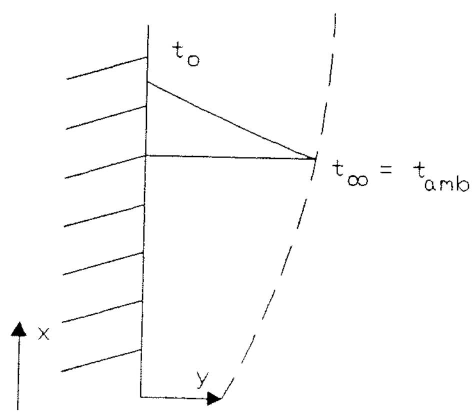


Figure 2-1B
Free convection thermal boundary layer

The only variable not specified is the temperature distribution of the surface. In general, the surface temperature can be specified as a power series (one dimensional).

$$t_o = t_\infty + \sum_n A_n x^n$$

for sufficient n to specify the surface temperature

Furthermore, if the power series can be reduced to the form

$$t_o - t_\infty = A x^n \tag{2-1}$$

then it can be shown that similarity solutions exist.

CASE 1

Vertical Flat Plate with a Constant Temperature Surface

It is quickly noticed, that the constant temperature surface can expressed in the form

$$t_o - t_\infty = Ax^n \quad \text{with } n = 0$$

or

$$t_o - t_\infty = A \quad \text{(a constant)}$$

Since the governing equations (continuity, momentum and energy) contain flow components in the x and y-directions (u,v), it is useful to transform the equations such that the dependence on u and v disappear. A technique that will do this is to define a stream function, ψ, such that

$$u = \frac{\delta\psi}{\delta y} \tag{2-2A}$$

$$v = \frac{\delta\psi}{\delta x} \tag{2-2B}$$

and to nondimensionalize the temperature distribution as

$$\theta = \frac{t - t_\infty}{t_o - t_\infty} \tag{2-3}$$

Substituting these into the governing equations yield

Momentum $$\rho\frac{\delta\psi}{\delta y}\frac{\delta^2\psi}{\delta x\delta y} - \rho\frac{\delta\psi}{\delta x}\frac{\delta^2\psi}{\delta y^2} = g\rho\beta\,(t_o - t_\infty)\,\theta + \mu\frac{\delta^3\psi}{\delta y^3} \tag{2-4}$$

Energy $$\rho c\frac{\delta\psi}{\delta y}\frac{\delta\theta}{\delta x} - \rho c\frac{\delta\psi}{\delta x}\frac{\delta\theta}{\delta y} + \theta\frac{\delta\psi}{\delta y}\frac{\delta t_o}{\delta x} = 0 \tag{2-5}$$

where the continuity equation drops out and $\frac{\delta t_o}{\delta x}$ can be written as an ordinary differential, i.e. $\frac{dt_o}{dx}$.

We now have a pair of partial differential equations which we want to reduce to ordinary differential equations. One technique for doing this is to define a similarity variable, η, and separate variables, i.e. assume the stream function is of the form, $\psi(x, \eta)$.

$$\eta = yH(x) \tag{2-6}$$

or

$$\psi(x, \eta) = v\, F(\eta)\, G(x) \tag{2-7}$$

By substituting equations (2-6) and (2-7) into equations (2-4) and (2-5), we find that

Momentum

$$G(x)\frac{dG(x)}{dx}\frac{dF(\eta)^2}{d\eta} - G(x)\frac{dG(x)}{dx}F(\eta)\frac{d^2F(\eta)}{d\eta^2} \tag{2-8}$$

$$= g\frac{\beta\,(t_o - t_\infty)\,\theta(\eta)}{\nu^2} + G(x)\frac{d^3F(\eta)}{d\eta^3}$$

Energy

$$3H(x)^2\, F(\eta)\frac{d\theta(\eta)}{\eta} + H(x)^2\theta(\eta) = 0 \tag{2-9}$$

If an appropriate choice for G(x) and H (x) is made, then these two equations will reduce to two ordinary differential equations with one independent variable, η. It can be shown that, if,

$$G(x) = 4\,(Gr_x/4)^{1/4} \tag{2-10}$$

and

$$H(x) = \frac{1}{x}\left(\frac{Gr_x}{4}\right)^{1/4} \tag{2-11}$$

then equations 2-8 and 2-9 can be written as

Momentum

$$\frac{d^3F(\eta)}{d\eta^3} + \theta(\eta) + 3F(\eta)\frac{d^2F(\eta)}{d\eta^2} - 2\left(\frac{dF(\eta)}{d\eta}\right)^2 = 0 \tag{2-12}$$

Energy

$$\frac{d^2\theta(\eta)}{d\eta^2} + 3PrF(\eta)\frac{d\theta(\eta)}{d\eta} = 0 \tag{2-13}$$

At this point, the boundary conditions must be written in terms of the variables F and θ.

Physical coordinates	Similarity coordinates
$u = 0$ at $y = 0$ $v = 0$ at $y = 0$ $u \rightarrow 0$ as $y \rightarrow \infty$ $t = t_o$ as $y = 0$ $t \rightarrow t_\infty$ as $y \rightarrow \infty$	$dF(\eta)/d\eta = 0$ at $\eta = 0$ $F(\eta) = 0$ at $\eta = 0$ $dF(\eta)/d\eta \rightarrow 0$ as $\eta \rightarrow \infty$ $\theta(\eta) = 1$ at $\eta = 0$ $\theta(\eta) \rightarrow 0$ as $\eta \rightarrow \infty$

It is possible to solve equations (2-12) and (2-13) numerically for the local Nusselt number

$$Nu_x = \frac{h_x x}{K} = -\frac{1}{\sqrt{2}} \frac{d\theta(\eta)}{d\eta} \bigg|_{\eta = 0} \cdot Gr_x^{1/4} \tag{2-14}$$

For variations in the Prandtl numbers, this result can be correlated by

$$Nu_x = \left(\frac{3}{4}\right) \left[\frac{2Pr}{5\,(1 + 2Pr^{1/2} + 2Pr)}\right]^{1/4} (Gr_x Pr)^{1/4} \tag{2-15}$$

This is simply the local Nusselt number. Oftentimes, it is desirable to find the overall heat transfer of the plate. This is done by integrating h_x over the surface of the plate.

$$\bar{h} = \frac{\int_0^L h_x dx}{\int_0^L dx} \tag{2-16}$$

$$= \frac{1}{L} \int_0^L -\frac{k}{x} \left(\frac{1}{\sqrt{2}} \frac{d\theta(\eta)}{d\eta} \bigg|_{\eta = 0} \bullet Gr_x^{1/4}\right) dx$$

$$= \frac{1}{L} - \left(\frac{k}{\sqrt{2}} \frac{d\theta(\eta)}{d\eta} \bigg|_{\eta = 0}\right) \left(\frac{g\beta\,(t_o - t_\infty)}{\nu^2}\right)^{1/4} \int_0^L x^{-1/4} dx$$

$$\bar{h} = \frac{1}{L} - \left(\frac{k}{\sqrt{2}} \frac{d\theta(\eta)}{d\eta} \Bigg|_{\eta = 0} \right) \left(\frac{g\beta\,(t_o - t_\infty)}{\nu^2} \right)^{1/4} \left[\frac{4}{3} x^{3/4} \right]_0^L$$

$$= \frac{4}{3} h_x = L$$

Thus, the overall heat convection coefficient is 4/3 the value of h at x equal to L.

CASE 2

Vertical Flat Plate with a Variable Surface Temperature

In equation (2-1), if $n \neq 0$ then the wall surface temperature is not constant. The same procedure in Case 1 is followed for reducing the governing equations to ordinary differential equations which are

Momentum
$$\frac{d^3F(\eta)}{d\eta^3} + \theta(\eta) + (n+3)\,F(\eta)\frac{d^2F(\eta)}{d\eta^2} - (2n+2)\left(\frac{dF(\eta)}{d\eta}\right)^2 = 0 \quad (2\text{-}17)$$

Energy
$$\frac{d^2\theta(\eta)}{d\eta} + Pr\left[(n+3)F(\eta)\frac{dF(\eta)}{d\eta} - 4n\frac{dF(\eta)}{d\eta}\theta(\eta)\right] = 0 \quad (2\text{-}18)$$

The boundary conditions are the same as before. One case of interest is the constant heat flux case. It can be shown that if

$$t_o - t_\infty = Ax^{1/5}$$

then a constant heat flux surface is present. The Nusselt number for this condition is given by

$$Nu_x = \left(\frac{Pr}{4 + 9Pr^{1/2} + 10Pr}\right)^{1/5}(Gr_x{}^*Pr)^{1/5} \quad (2\text{-}19)$$

where the Grashof number has been modified to contain the heat flux term, $\dot{q}_w$, and is given by

$$Gr_x^* = \frac{g\beta\dot{q}_w x^4}{\nu^2} \quad (2\text{-}20)$$

There also exists a large body of empirical correlations, most being of the form

$$\overline{Nu_f} = C\left(Gr_f Pr_f\right)^m \quad (2\text{-}21)$$

where the fluid properties are evaluated at the film temperature

$$T_f = \frac{T_o + T_\infty}{2} \quad (2\text{-}22)$$

SURFACE	$Gr_f Pr_f$	C	m
Vertical Plates	10^4 - 10^9	0.59	1/4
Horizontal	10^4 - 10^9	0.53	1/4
Horizontal Plates	10^7 - 10^{11}	0.15	1/3

Table 2-1

Constants found in equation (2-21) for isothermal surfaces

For horizontal flat that is not square in shape, the characteristic length is the average of the two sides, i.e.

$$L_m = \frac{L_1 + L_2}{2} \tag{2-23}$$

and for a circular disk, the characteristic length is 0.9d.

SECTION 3

FORCED CONVECTION

Forced convection past a boundary occurs when the fluid flow is induced by mechanical means such as a blower. The forced convection flow can be either laminar and turbulent in nature. The physical situation for forced convection over a vertical flat plate is shown in Figure 3-1.

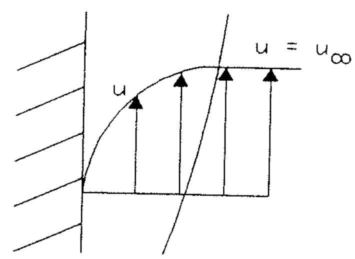


Figure 3-1

The analysis of forced convection heat transfer frequently involves a quantity called the Stanton number

$$St = Nu/RePr \tag{3-1}$$

which can be related to the friction coefficient, C_f, by

$$StPr^{2/3} = C_f/2 \tag{3-2}$$

and C_f, the local shear stress coefficient, can be found from the literature.

The solutions for the different geometries and flow regimes can be solved for by using the differential or integral forms of the continuity, momentum and energy equations. However, only the analytic and emperical results are given.

For semi-infinite vertical flat plate at constant temperature, the Nusselt number for laminar flow in a constant velocity field is given by

$$Nu_x = 0.332\, Pr^{1/3} Re^{1/2} \tag{3-3}$$

For turbulent flow over a semi-infinite vertical plate as above, the Stanton number is given by

$$St_x = \frac{0.0287\, Re_x^{-0.2}}{0.169 Re_x^{-0.1}(13.2Pr - 10.16) + 0.9} \qquad (3\text{-}4)$$

The previous two results are for constant temperature plates. If the surface temperature can be written as a power series, i.e. as

$$t_o - t_\infty = \sum_n A_n x^n \quad n = 0,1,2,... \qquad (3\text{-}5)$$

then the local heat flux can be determined at any given point on the plate. For the special case where the heat flux on the plate is constant, the Stanton number is given by

$$St_x Pr^{0.4} = 0.030\, Re_x^{-0.2} \qquad (3\text{-}6)$$

for turbulent boundary level.

If the flow is normal to a circular cylinder, the convection heat transfer coefficient can be determined from

$$Nu_d = C Re^n Pr^{1/3} \qquad (3\text{-}7)$$

where the constants C and n can be found in Table 3-1 and all fluid properties are evaluated at the film temperature.

Re_d	C	n
0.4 – 4	0.989	0.330
4 – 40	0.911	0.385
40 – 4000	0.683	0.466
4000 – 40,000	0.193	0.618
40,000 – 400,000	0.0266	0.805

Table 3-1 Constants for use in Equation 3-7